

(c)Prince Kochan's Production

問題Ⅰ. 次の式を因数分解しなさい。

$$\begin{aligned} \textcircled{1} \quad & 2a^2 + 14a + 24 \\ &= 2(a^2 + 7a + 12) \\ &= 2(a+3)(a+4) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & 5x^2 + 10x + 5 \\ &= 5(x^2 + 2x + 1) \\ &= 5(x+1)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & -2x^2 - 4x + 16 \\ &= -2(x^2 + 2x - 8) \\ &= -2(x-2)(x+4) \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad & -y^2 + 6y - 9 \\ &= -(y^2 - 6y + 9) \\ &= -(y-3)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad & 4x^2 + 8x + 4 \\ &= 4(x^2 + 2x + 1) \\ &= 4(x+1)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad & -3y^2 + 18y - 27 \\ &= -3(y^2 - 6y + 9) \\ &= -(y-3)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad & -x^2 - 5x + 36 \\ &= -(x^2 + 5x - 36) \\ &= -(x-4)(x+9) \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad & 8x^2 - 18 \\ &= 2(4x^2 - 9) \\ &= 2((2x)^2 - 9) \\ &= 2(2x+3)(2x-3) \end{aligned}$$

例3 $9x^2 + 24x + 15$
 $= (3x)^2 + 8 \times (3x) + 15$
 $= (3x+3)(3x+5)$

問題Ⅱ. 次の式を因数分解しなさい。

$$\begin{aligned} \textcircled{1} \quad & 4x^2 + 10x + 6 \\ &= (2x)^2 + 5 \times (2x) + 6 \\ &= (2x+2)(2x+3) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & 9x^2 + 24x + 16 \\ &= (3x)^2 + 8 \times (3x) + 16 \\ &= (3x+4)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & 9a^2 + 30a + 25 \\ &= (3a)^2 + 10 \times (3a) + 25 \\ &= (3a+5)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad & 4x^2 + 12x + 9 \\ &= (2x)^2 + 6 \times (2x) + 9 \\ &= (2x+3)^2 \end{aligned}$$

問題Ⅲ. 次の式を因数分解しなさい。

$$\begin{aligned} \textcircled{1} \quad & x^2 - 20xy + 100y^2 \\ &= (x-10y)(x-10y) \\ &= (x-10y)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & a^2 + 4ab + 4b^2 \\ &= (a+2b)(a+2b) \\ &= (a+2b)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & a^2 - 8ab + 16b^2 \\ &= (a-4b)(a-4b) \\ &= (a-4b)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad & 4x^2 + 12xy + 9y^2 \\ &= (2x)^2 + 6y \times (2x) + 9y^2 \\ &= (2x+3y)(2x+3y) \\ &= (2x+3y)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad & 25y^2 - 10xy + x^2 \\ &= x^2 - 10xy + 25y^2 \\ &= (x-5y)(x-5y) \\ &= (x-5y)^2 \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad & -12a^2 + 27b^2 \\ &= -3(4a^2 - 9b^2) \\ &= -3((2a)^2 - (3b)^2) \\ &= -3(2a+3b)(2a-3b) \end{aligned}$$

問題Ⅳ. 次の式を因数分解しなさい。

$$\begin{aligned} \textcircled{1} \quad & (a+b)^2 - 3(a+b) - 4 \\ & a+b = X \text{とおく} \\ & (a+b)^2 - 3(a+b) - 4 \\ &= X^2 - 3X - 4 \\ &= (X+1)(X-4) \\ &= (a+b+1)(a+b-4) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & (x+y)^2 - 10(x+y) + 16 \\ & x+y = X \text{とおく} \\ & (x+y)^2 - 10(x+y) + 16 \\ &= X^2 - 10X + 16 \\ &= (X-2)(X-8) \\ &= (x+y-2)(x+y-8) \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & (a+b)x + (a+b)(y+z) \\ & a+b = X \text{とおく} \\ & (a+b)^2 x + (a+b)(y+z) \\ &= Xx + X(y+z) \\ &= X(x+y+z) \\ &= (a+b)(x+y+z) \end{aligned}$$

$$\textcircled{4} \quad a(x-y) + bx - by$$

$$x-y = X \text{とおく}$$

$$a(x-y) + bx - by$$

$$= a(x-y) + b(x-y)$$

$$= aX + bX$$

$$= X(a+b)$$

$$= (x-y)(a+b)$$

$$\textcircled{5} \quad (x+y)^2 + 3(x+y) + 2$$

$$x+y = X \text{とおく}$$

$$(x+y)^2 + 3(x+y) + 2$$

$$= X^2 + 3X + 2$$

$$= (X+1)(X+2)$$

$$= (x+y+1)(x+y+2)$$

$$\textcircled{6} \quad (a+b)^2 - 2(a+b) + 1$$

$$a+b = X \text{とおく}$$

$$(a+b)^2 - 2(a+b) + 1$$

$$= X^2 - 2X + 1$$

$$= (X-1)^2$$

$$= (a+b-1)^2$$

$$\textcircled{7} \quad m(x-2y) + n(2y-x)$$

$$x-2y = X \text{とおく}$$

$$m(x-2y) + n(2y-x)$$

$$= m(x-2y) + n(-x+2y)$$

$$= m(x-2y) - n(x-2y)$$

$$= mX - nX$$

$$= X(m-n)$$

$$= (x-2y)(m-n)$$

$$\textcircled{8} \quad (x-3)^2 - 16$$

$$x-3 = X \text{とおく}$$

$$(x-3)^2 - 16$$

$$= X^2 - 16$$

$$= (X+4)(X-4)$$

$$= (x-3+4)(x-3-4)$$

$$= (x+1)(x-7)$$

(別解)

$$(x-3)^2 - 16$$

$$= x^2 - 6x + 9 - 16$$

$$= x^2 - 6x - 7$$

$$= (x+1)(x-7)$$

$$\textcircled{9} \quad a(x-y) - bx + by$$

$$x-y = X \text{とおく}$$

$$a(x-y) - bx + by$$

$$= a(x-y) - b(x-y)$$

$$= aX - bX$$

$$= X(a-b)$$

$$= (x-y)(a-b)$$

$$\textcircled{10} \quad ax - 3a - x + 3$$

$$x-3 = X \text{とおく}$$

$$ax - 3a - x + 3$$

$$= a(x-3) - (x-3)$$

$$= aX - X$$

$$= X(a-1)$$

$$= (x-3)(a-1)$$

問題V 次の式を因数分解しなさい。

$$\textcircled{1} \quad x^3 + x^2 + x + 1$$

$$= x^2(x+1) + (x+1)$$

$$= (x+1)(x^2+1)$$

$$\textcircled{2} \quad x^3 + x^2 - x - 1$$

$$= x^2(x+1) - (x+1)$$

$$= (x+1)(x^2-1)$$

$$= (x+1)(x+1)(x-1)$$

$$= (x+1)^2(x-1)$$

$$\textcircled{3} \quad x^2 + xy + 4x + 4y$$

$$= x(x+y) + 4(x+y)$$

$$= (x+y)(x+4)$$

$$\textcircled{4} \quad x^2 - y^2 + 2x - 2y$$

$$= (x+y)(x-y) + 2(x-y)$$

$$= (x-y)(x+y+2)$$

$$\textcircled{5} \quad x^2 - 3xy + 3x - 9y$$

$$= x(x-3y) + 3(x-3y)$$

$$= (x-3y)(x+3)$$

$$\textcircled{6} \quad a^2 - ab - bc + ac$$

$$= a^2 - ab + ac - bc$$

$$= a(a-b) + c(a-b)$$

$$= (a-b)(a+c)$$

$$\textcircled{7} \quad x^4 - x^2 + x - 1$$

$$= (x^2)^2 - x^2 + x - 1$$

$$= x^2(x^2-1) + x-1$$

$$= x^2(x+1)(x-1) + (x-1)$$

$$= (x-1)(x^2(x+1) + 1)$$

$$= (x-1)(x^3 + x^2 + 1)$$

問題VI 次の式を因数分解しなさい。

$$\textcircled{1} \quad x^2 - y^2 - 6y - 9$$

$$= x^2 - (y^2 + 6y + 9)$$

$$= x^2 - (y+3)(y+3)$$

$$= x^2 - (y+3)^2$$

$$= \{x+(y+3)\} \{x-(y+3)\}$$

$$= (x+y+4)(x-y-3)$$

$$\textcircled{2} \quad x^2 - y^2 + 6y - 9$$

$$= x^2 - (y^2 - 6y + 9)$$

$$= x^2 - (y-3)(y-3)$$

$$= x^2 - (y-3)^2$$

$$= \{x+(y-3)\} \{x-(y-3)\}$$

$$= (x+y-3)(x-y+3)$$